

ABSTRACT

This study deals with the effect of variable viscosity and Prandtl number on the steady, laminar free convection flow (of iso-propanol) past a vertical porous flat plate. The coupled nonlinear partial differential equations governing the self-similar flow have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization technique. Numerical results indicate that variable viscosity (and Prandtl number) has a major role on skin friction and heat transfer parameters as well as velocity and temperature fields. Further, it is observed that the effect of variable fluid properties plays a significant role in the control of laminar boundary layer. The present analysis reveals the fact that when the working fluid is sensitive to the temperature, the effect of variable viscosity and Prandtl number has to be taken into the consideration in order to predict the skin friction and heat transfer rate, accurately.

Keywords: *Skin friction, Heat transfer, Temperature-dependent Viscosity, Velocity, Temperature.*
MSC 2010 Codes – 76M20, 76N20, 76R1

I. INTRODUCTION

Applications of heat transfer are generally based on the constant physical properties of the ambient fluid in fluid dynamics research. However, it is known that these properties may change with temperature, especially the fluid viscosity and hence, the Prandtl number. Numerous researchers have studied the effect of variable viscosity on different geometries under various situations [1-7].

Free convection boundary layer flows frequently encountered in environmental and engineering devices. Abundant literature is available on the topic of the laminar boundary layer flow over a porous vertical plate with suction and injection, having wide range of engineering applications. In fact, the case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [8]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [9], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection). The present study is undertaken to investigate the effect of variable viscosity and Prandtl number on the free convection boundary layer flow (of iso-propanol) over a vertical porous plate with suction.

II. GOVERNING EQUATIONS

We consider a semi-infinite porous plate, which is placed vertical in a quiescent fluid (iso-propanol) of infinite extent maintained at a uniform temperature. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge (origin) in the stream wise direction and y is measured normal to the surface of the plate. Indeed, the flow is assumed to be in the x -direction i.e., along the vertical plate in the upward direction and the y -axis is taken to be normal to the plate.

The fluid properties are assumed to be isotropic and constant except for the fluid viscosity. The temperature difference between the surface of the plate (T_w) and the ambient fluid (T_∞) is taken to be small ($< 60^\circ\text{C}$). In the range of temperature (T) considered (i.e. $0-60^\circ\text{C}$), the variation of both density (ρ) and specific heat (c_p) of iso-



propanol with temperature, is less than 1% (See Table I) and hence they are taken as constants. However, since the variation of thermal conductivity (k) and viscosity (μ) [and hence the Prandtl number (Pr)] with temperature is quite significant, the viscosity (μ) and Prandtl number are assumed to vary as an inverse linear function of temperature:

$$\mu = 1/(b_1 + b_2 T) \quad (1)$$

$$\text{Pr} = 1/(c_1 + c_2 T) \quad (2)$$

where

$$b_1 = 170.147, b_2 = 14.518, c_1 = 0.1198 \text{ and } c_2 = 0.0057 \quad (3)$$

Table I: Values of thermo-physical properties of Iso-propanol at different temperature [10].

Temperature (T) ($^{\circ}\text{C}$)	Density (ρ) (gr./c m ³)	Specific heat (c_p) (J $\times 10^7$ /kg $^{\circ}\text{K}$)	Thermal conductivity (k) (erg $\times 10^5$ /c m.s- $^{\circ}\text{K}$)	Viscosity (μ) (gr. $\times 10^{-2}$ /cm-s)	Prandtl number (Pr)
0	0.804	2.330	0.141	0.4514	7.46
10	0.795	2.443	0.138	0.3244	5.74
20	0.786	2.562	0.136	0.2381	4.48
30	0.777	2.683	0.134	0.1782	3.56
40	0.767	2.807	0.132	0.1351	2.88
50	0.758	2.931	0.129	0.105	2.38
60	0.748	3.056	0.127	0.0824	1.98

The relation (1) and (2) are reasonably holds good approximations for liquids such as iso-propanol, particularly for small wall and ambient temperature differences. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow. Under the above-mentioned assumptions with Boussinesq's approximation, the boundary layer equations governing the steady, two-dimensional flow are [11]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T - T_{\infty}) + \frac{1}{\rho_{\infty}} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (5)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho_{\infty}} \frac{\partial}{\partial y} \left[\left(\text{Pr}^{-1} \mu \frac{\partial T}{\partial y} \right) \right] \quad (6)$$

The initial and boundary conditions are

$$\begin{aligned} x = 0, y > 0; u = v = 0, T = T_w \\ x > 0; y = 0; u = 0, v = -v_0 \text{ (for suction)}, \\ x > 0; y = 0; u = 0, v = +v_0 \text{ (for injection)}, \\ y \rightarrow \infty; x > 0; u = 0, T = T_{\infty} \end{aligned} \quad (7)$$

Introducing the following transformations

$$u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x}; \quad \psi = \frac{v_0^2 g \beta (T_w - T_\infty) \xi^3}{V_0^3} \left[f(\eta, \xi) \pm \frac{\xi}{4} \right]$$

$$T = T_\infty + (T_w - T_\infty) G(\eta, \xi); \quad \eta = \frac{V_0 y}{v \xi}; \quad \xi = V_0 \left[\frac{4x}{v^2 g \beta (T_w - T_\infty)} \right]^{1/4} \quad (8)$$

to Eqns.(4) – (6), we see that the continuity Eq.(4) is identically satisfied and Eqns.(5) – (6) reduces, respectively, to

$$(NF')' + 3fF'' - 2F^2 \pm \xi F' + G = \xi (F F_\xi - F' f_\xi) \quad (9)$$

$$(NPr^{-1}G')' + 3fG' \pm \xi G' = \xi (FG_\xi - G' f_\xi) \quad (10)$$

where

$$u = \frac{V_0^2 4x}{v \xi^2} F; \quad v = -\frac{V_0}{\xi} (3f + \xi f_\xi - \eta F \pm \xi)$$

$$f = \int_0^\eta F d\eta; \quad N = \left(\frac{\mu}{\mu_\infty} \right) = \frac{b_1 + b_2 T_\infty}{b_1 + b_2 T} = \frac{1}{1 + a_1 G},$$

$$Pr = \frac{1}{c_1 + c_2 T} = \frac{1}{a_2 + a_3 G}, \quad a_1 = \left(\frac{b_2}{b_1 + b_2 T_\infty} \right) \Delta T_w,$$

$$a_2 = c_1 + c_2 T_\infty, \quad a_3 = c_2 \Delta T_w, \quad \Delta T_w = (T_w - T_\infty) \quad (11)$$

It is noted here that the upper and lower signs in Eqns. (9) and (10) is taken thought for suction and injection, respectively. The present study, however, restricted to the case of suction only.

The transformed boundary conditions are:

$$\begin{aligned} F = 0; \quad G = 1 \quad \text{at} \quad \eta = 0 \\ F = 0; \quad G = 0 \quad \text{as} \quad \eta \rightarrow \infty \end{aligned} \quad (12)$$

The local skin friction parameter and heat transfer parameter can be expressed as

$$\tau_w = \frac{V_0}{g \beta (T_{w0} - T_\infty)} \left(\frac{\partial u}{\partial y} \right)_{y=0} = \xi (F')_{\eta=0} \quad (13)$$

$$Q = \frac{v}{V_0 (T_{w0} - T_\infty)} \left(\frac{\partial T}{\partial y} \right)_{y=0} = -\frac{1}{\xi} (G')_{\eta=0} \quad (14)$$

Here, u and v are velocity components in x and y -directions respectively; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ and η are transformed co-ordinates; ψ and f are the dimensional and dimensionless stream functions respectively; Pr is the Prandtl number; $a_1, a_2, a_3, b_1, b_2, c_1, \& c_2$ are constants; g is the gravitational acceleration; β is the coefficient of thermal expansion; w and ∞ denote conditions at the edge of the boundary layer on the wall and in the free stream respectively, and prime ($'$) denotes derivatives with respect to η .

III. RESULTS AND DISCUSSION

The set of partial differential Eqns.(9) and (10) along with the boundary conditions (12) has been solved numerically employing an implicit finite difference scheme with a quasilinearization technique [11,12]. The detailed description is omitted here to conserve the space. In order to assess the accuracy of the numerical method which we have used, the skin friction and heat transfer parameters (τ_w, Q) for suction have been obtained by solving Eqs.(9) and (10) for constant viscosity [N=1] case, taking Pr=1.0, and compared with those of Merkin [9]. Our results are found to be in good agreement with those of [9], validating the accuracy of the numerical method used in the present study. The computed results for variable fluid properties have been presented in the graphical form and analyzed.

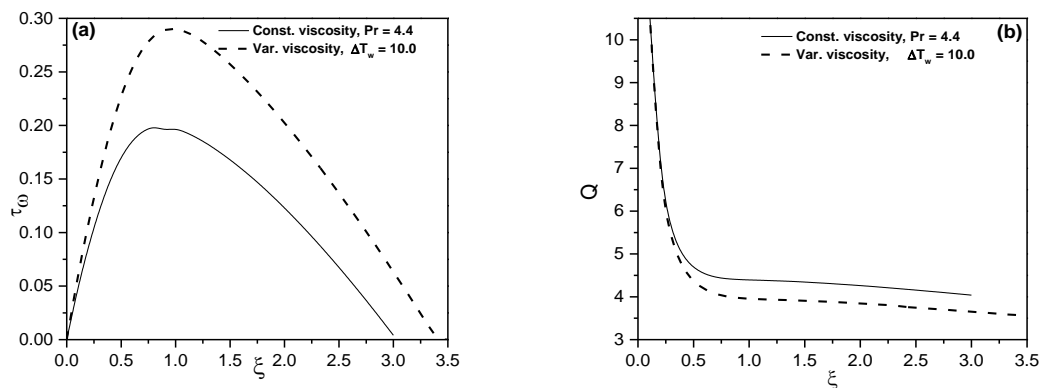


Fig. 1. Variation of (a) skin friction and (b) heat transfer parameters along stream-wise directions

Figure 1 describes the variation of skin friction (τ_w) and heat transfer parameters (Q) with the stream wise coordinate ξ , in the presence of both variable fluid properties [$T_\infty = 25.7^\circ\text{C}$, $\Delta T_w = 10.0$] and constant fluid properties [$N = 1$ and $Pr = 4.4$], in the presence of suction. It is observed from Fig.1(a) that skin friction (τ_w) increases from zero to a maximum value in a certain range of ξ say $\xi = 1.0$, and then decreases as ξ further increases. It is also observed that the effect of variable fluid properties is to increase the skin friction and to decrease the heat transfer. In fact, τ_w for variable fluid properties differs from that of constant fluid properties by about 38.9% at $\xi = 1.0$ and about 40.49% at $\xi = 2.0$ [Fig.1(a)]. On the other hand, the percentage of difference between constant and variable fluid properties, in the case of Q [Fig.1(b)] is 11.10% at $\xi = 1.0$ and about 10.82% at $\xi = 2.0$. Further, it is important to observe that in the case of variable fluid properties, the point of zero skin friction moved downstream as compared to constant fluid properties. Indeed, the point of zero skin friction occurs at $\xi = 3.1$ for constant fluid properties whereas it is visible at $\xi = 3.4$ [Fig.1(a)] for variable fluid properties. This indicates the controlling of laminar boundary layer avoiding boundary layer separation.

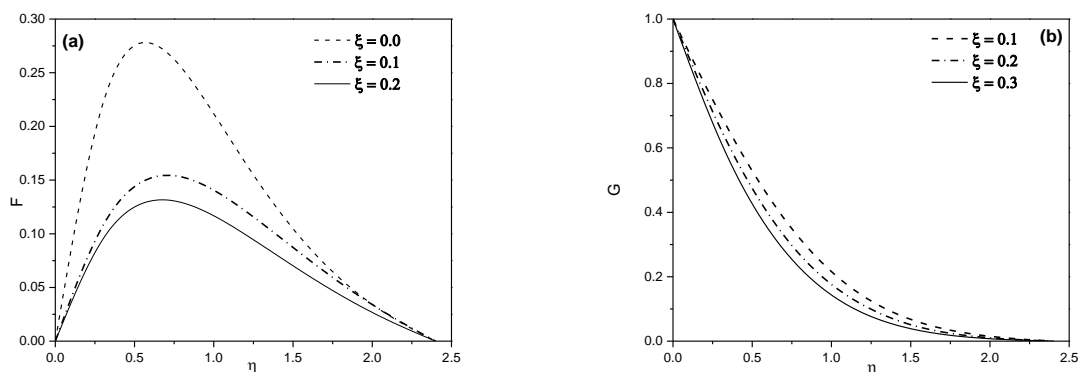


Fig. 2. behaviour of (a) Velocity and (b) Temperature profiles at different stream-wise locations

The relevant velocity (F) and temperature (G) profiles are shown in Fig.2, for the case of variable fluid properties. It is observed that the thickness of momentum boundary layer increases with the increase of streamwise coordinate (ξ) [Fig.2(a)], which results in the velocity of the fluid inside the boundary layer. On the other hand, the thermal boundary layer thickness decreases as ξ increases, enhancing the temperature inside the boundary layer [Fig.2(b)].

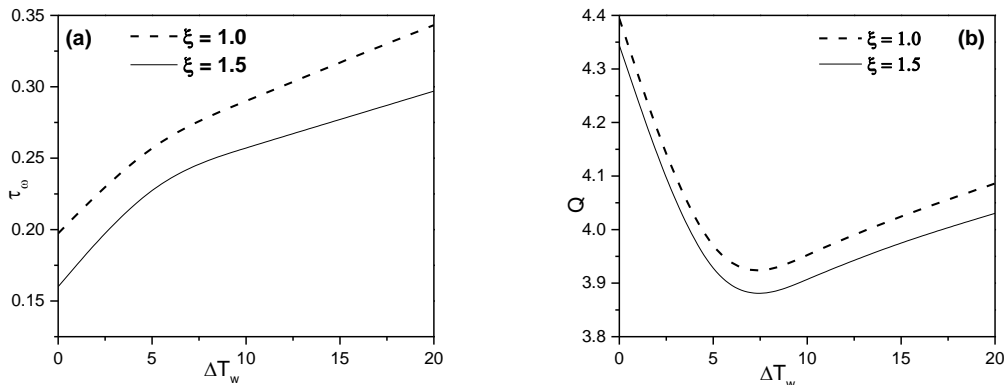


Fig. 3. Effect of ΔT_w on (a) Skin friction and (b) Heat transfer parameters at stream-wise locations

The variation of skin friction (τ_w) and heat transfer parameters (Q) with ΔT_w (difference temperature between the surface of the plate and the ambient fluid,) is portrayed in Fig.3, to see the effect of variation of viscosity and Prandtl number with temperature across the boundary layer. Since $T_\infty = 25.7^\circ\text{C}$, the maximum value of ΔT_w taken is 20°C so as to keep the temperature within the allowed value ($< 60^\circ\text{C}$), during numerical computations. In these figures, we observe that τ_w increases with the increase of ΔT_w along the stream wise locations. However, Q is found decrease up to $\Delta T_w = 5^\circ\text{C}$ and later increases beyond $\Delta T_w > 5^\circ\text{C}$. Also, Q is found to decrease with the increase of ξ . Further, it is observed that the heat transfer decreases by about 1.179% when $\Delta T_w = 10^\circ\text{C}$ and, about 1.227% when $\Delta T_w = 15^\circ\text{C}$, at $\xi = 1$ and $\xi = 1.5$, respectively. On the other hand, the skin friction decreases by about 12.67% when $\Delta T_w = 10^\circ\text{C}$ and, about 14.435% when $\Delta T_w = 15^\circ\text{C}$, at same stream wise locations of the vertical plate. It is clear from these figures that temperature-dependent viscosity as well as Prandtl number plays a key role in providing strong coupling of energy and momentum transfer, exposing their supremacy within the laminar boundary layer region.

IV. CONCLUSIONS

The steady, laminar iso-propanaol boundary layer flow past a vertical porous flat plate is numerically investigated assuming both viscosity and Prandtl number as linear inverse functions of temperature. The computed results show that the flow/temperature fields, skin friction and heat transfer characteristics are significantly affected by the temperature-dependent viscosity and Prandtl number. From the present study, it is concluded that the effect of variable fluid properties plays a significant role in the control of laminar boundary layer.

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